

Evidence for color fluctuations in the nucleon in high-energy scattering

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We study quantum fluctuations of the nucleon's parton densities by combining QCD factorization for hard processes with the notion of cross section fluctuations in soft diffraction. The fluctuations of the small- x gluon density are related to the ratio of inelastic and elastic vector meson production in ep scattering. A simple dynamical model explains the HERA data and predicts the x - and Q^2 -dependence of the ratio. In $pp/\bar{p}p$ scattering, fluctuations enhance multiple hard processes (but cannot explain the Tevatron CDF data), and reduce gap survival in central exclusive diffraction.

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Hard processes in high-energy ep and $pp/\bar{p}p$ scattering probe nucleon structure at a resolution scale where it can be described in terms of the quark and gluon degrees of freedom of QCD. Essential in the analysis of such processes is the method of factorization, by which the amplitude or cross section is separated into a short-distance quark/gluon subprocess, calculable in perturbative QCD, and the distributions of the partons in the initial and final hadrons, governed by long-distance, non-perturbative interactions. Inclusive scattering experiments probe the longitudinal momentum densities of the partons. Measurements of hard exclusive processes in ep scattering reveal information also about their spatial distribution in the transverse plane (generalized parton distributions, or GPDs). Such experiments can eventually provide us with a full 3-dimensional image of the quark/gluon single-particle structure of the nucleon.

From the perspective of many-body physics the parton densities represent average characteristics of the nucleon, reflecting the quantum-mechanical average over configurations in the nucleon wave function of different size, number of particles, *etc.* Equally fundamental are the fluctuations around the average value, which generally provide information about the nature of the interaction in the system. In the case of the quark and gluon densities the fluctuations are related to variations of the size and intensity of the long-wavelength color fields inside hadrons — information crucial for understanding hadron structure in terms of non-perturbative QCD. An interesting question is which experimental observables could reveal such “color fluctuations” inside hadrons.

To see how this problem might be approached, it is worthwhile to recall some facts about soft diffractive hadron-hadron scattering at high energies and low momentum transfer, t . In such processes the hadrons can be represented as a superposition of states which diagonalize the T -matrix and experience only individual attenuation (“diffractive eigenstates”). Elastic diffraction then results from uniform attenuation of these components, while inelastic diffraction is caused by different

attenuation, which destroys the coherence of the original hadronic wave function. This idea was originally formulated using the language of the eikonal approximation appropriate for non-relativistic scattering [1, 2]. Later a more general formulation was developed, consistent with the fundamental principles of relativistic quantum field theory (causality, energy-momentum conservation). It uses the concept of the cross section distribution, $P(\sigma)$, which describes the probability for the hadrons to scatter in a configuration with given cross section [2, 3]. It is normalized as $\int d\sigma P(\sigma) = 1$, and its variance is related to the ratio of inelastic and elastic soft diffractive cross sections at $t = 0$,

$$\omega_\sigma \equiv \frac{\langle \sigma^2 \rangle - \langle \sigma \rangle^2}{\langle \sigma \rangle^2} = \left[\frac{d\sigma_{\text{inel}}}{dt} / \frac{d\sigma_{\text{el}}}{dt} \right]_{t=0}^{\text{soft diff}}, \quad (1)$$

where the brackets denote the average with the distribution $P(\sigma)$. In particular, this formulation allows one to incorporate color transparency (the vanishing of the interaction of small-size configurations) in the $\sigma \rightarrow 0$ behavior of $P(\sigma)$, and the approach to the black-disk regime (unitarity limit) at high energies; both are fundamental predictions of QCD. Detailed phenomenological studies of various diffractive phenomena involving proton and nuclear targets have shown the usefulness of this approach and determined the properties of $P(\sigma)$ [4, 5]. At pp energies $\sqrt{s} \sim 20 \text{ GeV}$ the variance is $\omega_\sigma \sim 0.25$, indicating sizable fluctuations of the interacting configurations in soft processes.

In this Letter we propose to study color fluctuations in hadrons by combining QCD factorization for hard processes with the notion of cross section fluctuations in soft diffractive processes. We introduce the concept of a configuration-dependent parton density and follow its implications for various types of high-energy scattering experiments with hard processes. Our investigation proceeds in three stages. First, we relate the fluctuations of the gluon density to the ratio of inelastic to elastic hard diffraction in ep scattering (HERA, future Electron-Ion Collider, or EIC). Second, we propose a simple model of

color fluctuations in the nucleon to illustrate and quantify our results. Third, we discuss the implications of color fluctuations for $pp/\bar{p}p$ collisions with multiple hard processes, and for rapidity gap survival in exclusive diffractive pp scattering (Tevatron, LHC). The effects described here are not included in present Monte-Carlo generators for pp collisions. A more detailed account of our studies will be given elsewhere [6].

Consider diffractive production of vector mesons in ep scattering at $Q^2 \gtrsim \text{few GeV}^2$, $\gamma_L^* + p \rightarrow V + X$, where the proton may remain intact or dissociate into hadronic states X . The initial proton state can be expanded in a set of partonic states characterized by the number of partons and their transverse positions, summarily labeled as $|n\rangle$: $|p\rangle = \sum_n a_n |n\rangle$. Each configuration n has a definite gluon density $G(x, Q^2|n)$, given by the expectation value of the twist-2 gluon operator in the state $|n\rangle$, and the overall gluon density in the proton is

$$G(x, Q^2) = \sum_n |a_n|^2 G(x, Q^2|n) \equiv \langle G \rangle. \quad (2)$$

Because the partonic states appear “frozen” on the typical timescale of the hard scattering process, one can use QCD factorization to calculate the amplitude for vector meson production configuration by configuration. It is (up to small calculable corrections) proportional to the gluon density in that configuration [7]. An essential point is now that in the leading-twist approximation the hard scattering process attaches to a single parton, and, moreover, does not transfer momentum to that parton; it thus does not change the partonic state $|n\rangle$. Making use of the completeness of partonic states, we find that the elastic ($X = p$) and total diffractive (X arbitrary) cross sections are proportional to

$$(d\sigma_{\text{el}}/dt)_{t=0} \propto [\sum_n |a_n|^2 G(x, Q^2|n)]^2 \equiv \langle G \rangle^2, \quad (3)$$

$$(d\sigma_{\text{diff}}/dt)_{t=0} \propto \sum_n |a_n|^2 [G(x, Q^2|n)]^2 \equiv \langle G^2 \rangle. \quad (4)$$

For the cross section of inelastic diffraction $\sigma_{\text{inel}} = \sigma_{\text{diff}} - \sigma_{\text{el}}$ we thus obtain

$$\omega_g \equiv \frac{\langle G^2 \rangle - \langle G \rangle^2}{\langle G \rangle^2} = \left[\frac{d\sigma_{\text{inel}}}{dt} \Big/ \frac{d\sigma_{\text{el}}}{dt} \right]_{t=0}^{\gamma_L^* p \rightarrow V X}. \quad (5)$$

This model-independent relation allows one to infer the fluctuations of the gluon density from the observable ratio of inelastic and elastic diffractive vector meson production. It can be easily generalized to a large variety of hard processes such as $\gamma_L^* + T \rightarrow 2\pi$ (two jets) + T , or Υ production in ultraperipheral pp collisions at LHC [8].

Generally, we expect ω_g to be a weak function of Q^2 at fixed x (approximate scaling), as the gluon density depends only logarithmically on Q^2 . The x -dependence of ω_g at fixed Q^2 is difficult to infer from first principles; it depends on the “color flow” in the nucleon wave function, *i.e.*, how the small- x parton densities change with the configuration of the large- x constituents.

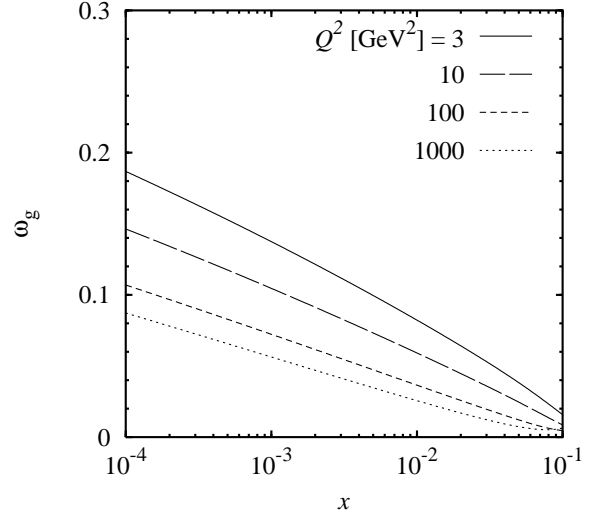


FIG. 1: The variance of fluctuations of the proton’s gluon density, ω_g , as a function of x for several values of Q^2 , as obtained from the scaling model, Eq. (6), and a phenomenological parametrization of the gluon density.

To estimate the variance of the gluon fluctuations at small x , and to study their implications in other hard scattering processes, we propose here a simple model based on two assumptions: (a) At moderate energies ($\sqrt{s} \sim 20$ GeV) the hadronic cross section of a configuration is proportional to the transverse area occupied by the color charges in that configuration, $\sigma \propto R_{\text{config}}^2$; (b) the normalization scale of the parton density changes proportionally to the size of the configuration, $\mu^2 \propto R_{\text{config}}^{-2} \propto \sigma^{-1}$. Assumption (b) is similar to the “nucleon swelling” model of the EMC effect [9] and implies a simple scaling relation for the σ -dependent gluon density:

$$G(x, Q^2|\sigma) = G(x, \xi(Q^2)), \quad \xi(Q^2) \equiv (\sigma/\langle\sigma\rangle)^{\alpha_s(Q_0^2)/\alpha_s(Q^2)}, \quad (6)$$

where $Q_0^2 \sim 1 \text{ GeV}^2$. Assumption (a) then allows us to compute the configuration average using the phenomenological cross section distribution found in Ref. [4]. Figure 1 shows the result for the variance of the gluon density in this model. At small x and low Q^2 it reaches values comparable to the variance of soft cross section fluctuations, ω . Note that our evolution-based model of gluon fluctuations applies primarily to small x ($\ll 0.1$); at larger x non-perturbative correlations not included here may become important. Present experimental data on the cross section ratio in Eq. (5) are very limited. The value $\omega_g \sim 0.15 - 0.2$ for $Q^2 = 3 \text{ GeV}^2$ and $x \sim 10^{-4} - 10^{-3}$ obtained in our model is consistent with the HERA data on vector meson production, where the effective scale is $Q_{\text{eff}}^2 \sim 2 - 4 \text{ GeV}^2$. The data also indicate weak dependence of the ratio on Q^2 and the vector meson mass; however, the limited Q^2 range and the lack of

dedicated studies do not allow us to test our model predictions in more detail. Future measurements with LHC [8] and EIC could significantly improve the situation.

Correlations between fluctuations of the parton densities and the soft-interaction strength have numerous potential implications for high-energy $pp/\bar{p}p$ collisions with hard processes. One example is the relative probability of double binary parton-parton collisions (see Fig. 2a), defined as the ratio

$$\frac{\frac{d\sigma(x_1, x_2; x_3, x_4)}{d\Omega_{12} d\Omega_{34}}}{\frac{d\sigma(x_1, x_2)}{d\Omega_{12}} \frac{d\sigma(x_3, x_4)}{d\Omega_{34}}} = \frac{f(x_1, x_3)f(x_2, x_4)}{\sigma_{\text{eff}} f(x_1)f(x_2)f(x_3)f(x_4)}, \quad (7)$$

where Ω_{12} *etc.* are the variables characterizing the observed dijets (or photons), and $f(x_1)$ and $f(x_1, x_3)$ *etc.* are the single and double parton densities, respectively (we suppress the dependence on the scale). The effective cross section σ_{eff} is a measure of transverse correlation between partons. The FNAL CDF experiment measured a value of $\sigma_{\text{eff}} = 14.5 \pm 1.7_{-2.3}^{+1.7}$ mb in 3-jet + γ events [10], a factor of ~ 3 smaller than the naive estimate based on a uniform distribution of partons with the e.m. radius of the nucleon. Proper account of the transverse geometry, using gluon GPDs based on J/ψ photoproduction data, reduces the discrepancy to a factor of ~ 2 [11]. An interesting question is whether fluctuations of the sizes of the interacting configurations and the gluon density could explain the remaining discrepancy.

Elementary geometric arguments show that in the mean-field approximation (*i.e.*, no fluctuations)

$$\sigma_{\text{eff}}^{-1}(\text{mean field}) = \int d^2b P_{12}(b) P_{34}(b), \quad (8)$$

where $b \equiv |\mathbf{b}|$ is the pp impact parameter, and $P_{12}(b)$ describes the b -dependence of the probability to have two partons with x_1 and x_2 collide in the pp collision,

$$P_{12}(b) \equiv \int d^2\rho_1 \int d^2\rho_2 \delta^{(2)}(\mathbf{b} - \boldsymbol{\rho}_1 + \boldsymbol{\rho}_2) \times F(x_1, \rho_1) F(x_2, \rho_2), \quad (9)$$

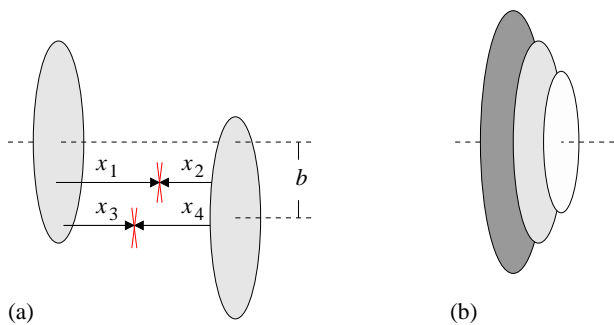


FIG. 2: (a) Double hard scattering in high-energy pp -collisions. (b) Schematic illustration of fluctuations. In the model used here [*cf.* Eq. (6)], configurations with larger size have larger gluon density.

with $\int d^2b P_{12}(b) = 1$ (a similar definition applies to P_{34}). Here $F(x, \rho)$ is the normalized transverse spatial distributions of the partons obtained from the GPD. To estimate the effect of fluctuations, we now apply the same reasoning to the instantaneous configurations in the colliding protons, and let both the transverse size and the gluon density in these configurations fluctuate as described by our model [*cf.* Eq. (6)]. We assume that the transverse size of the configurations is proportional to σ , *i.e.*, the spatial distribution of partons is $F(x, \rho|\sigma) = \lambda^{-1} F(x, \lambda^{-1/2}\rho)$, with $\lambda \equiv \sigma/\langle\sigma\rangle$ (see Fig. 2b). With these assumptions we obtain

$$\begin{aligned} \sigma_{\text{eff}}^{-1}(\text{fluct}) &= \langle\langle \phi(x_1|\sigma_1) \phi(x_2|\sigma_2) \phi(x_3|\sigma_1) \phi(x_4|\sigma_2) \\ &\quad \times \int d^2b P_{12}(b|\sigma_1, \sigma_2) P_{34}(b|\sigma_1, \sigma_2) \rangle\rangle_{12}, \\ \phi(x_1|\sigma_1) &\equiv f(x_1|\sigma_1) / \langle f(x_1|\sigma_1) \rangle_1 \text{ etc.}, \end{aligned} \quad (10)$$

where the double brackets denote the average over the σ -distributions for both protons.

Because in multijet production at the Tevatron the typical x -values of the partons are large (~ 0.1), fluctuations of the parton densities are much smaller than in vector meson production at HERA (see Fig. 1). The dominant effect in Eq. (10) thus comes from fluctuations of the sizes of the interacting configurations. Keeping only the latter, we obtain a simple analytic result by expanding in leading order in the variance ω_σ :

$$\sigma_{\text{eff}}(\text{fluct}) = (1 - \omega_\sigma/2) \sigma_{\text{eff}}(\text{mean field}); \quad (11)$$

numerical studies show that higher-order corrections are negligible in practice. One sees that size fluctuations indeed reduce σ_{eff} , because of the disproportionate enhancement of multiple hard processes in small-size configurations. However, the reduction in our model is found to be only of the order 10 – 15%, which cannot account for the discrepancy with the CDF value. This indicates that other dynamical mechanisms must be responsible for the enhancement of multi-parton collisions, *e.g.* local transverse correlations between partons as suggested by a “constituent quark” picture of the nucleon [12].

We note that much stronger reduction of σ_{eff} was obtained [13] in a simplified multi-channel eikonal model, where all hadronic diffractive states are represented by a single channel [14]. In such a model, to fit the available data on elastic, inelastic, single and double diffractive cross sections, one needs to enhance the strength of the Pomeron coupling between diffractive eigenstates of smaller radii. In the present model this would correspond to an increase, rather than a decrease, of the parton density for small σ , which is the origin of the different results.

Another class of processes affected by fluctuations is central exclusive diffraction, $pp \rightarrow p + H + p$ (H = dijet, heavy quarkonium, Higgs boson), widely discussed as a possible channel for the Higgs boson search at the LHC. Of interest here is the rapidity gap survival (RGS) probability, S^2 , measuring the suppression of the cross section

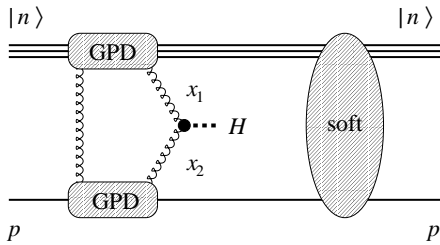


FIG. 3: Fluctuations in central exclusive diffraction $pp \rightarrow p + H + p$. One of the protons is treated as a superposition of partonic states $|n\rangle$, which undergo diffractive scattering with the other proton (small fluctuation approximation).

resulting from the requirement of no inelastic soft spectator interactions. This quantity has extensively been studied using models based on eikonalized Pomeron exchange [15]. A recent analysis in a partonic approach with account of the transverse geometry (GPDs) found that the suppression of the diffractive cross section results mainly from the elimination of pp collisions at small impact parameters, in which there is a high probability of inelastic interactions [16]. This result was obtained in the mean-field approximation, in which the GPDs and the soft-interaction strength are taken at their average values, with no correlations between them. (The approach of Ref. [15] gives results comparable to the mean-field approximation and also ignores such correlations.)

To estimate the effects of correlations, we follow the approach described above and allow for configuration dependence of both the gluon GPDs (*i.e.*, the gluon density and the radius of the transverse distribution) and the soft-interaction strength. The latter we model phenomenologically, using data on the energy dependence of $\langle\sigma\rangle$ and ω_σ in soft hadron-hadron scattering [6]. At small x the dominant effect comes from the correlation of fluctuations of the gluon density with the soft-interaction strength. Treating the fluctuations as a small correction to the mean-field result we have

$$S^2(\text{fluct}) = (1 + 4\epsilon) S^2(\text{mean field}), \quad (12)$$

where ϵ is the correction obtained if one of the protons fluctuates (see Fig. 3); the factor 4 counts the number of protons in the diffractive amplitude and its complex conjugate. We find $\epsilon \approx -0.07$ (-0.04) for production of a system with mass $M_H = 10$ (100) GeV at $\sqrt{s} = 2$ TeV (Tevatron), resulting in a reduction of the RGS probability by a factor ~ 0.7 (0.85). The sign reflects the fact that smaller configurations with higher survival probability have a lower density of small- x gluons in our model (see Fig. 2b). We note that at higher energies (LHC) the onset of the black-disk regime in hard interactions causes another, more substantial reduction of the RGS probability relative to the mean-field result [16].

Fluctuations also affect the final-state transverse momentum dependence of the cross section for central ex-

clusive diffraction, because they change the size of the dominant interacting configurations. Our findings suggest that the p_T distribution at RHIC and Tevatron energies is narrower than given by the mean-field approximation — a prediction which could be tested by future measurements of diffraction at RHIC.

In sum, the study of quantum-mechanical fluctuations of the quark/gluon densities is the natural next step in the exploration of nucleon structure in QCD, following the mapping of the longitudinal momentum and transverse spatial distributions of partons. Detailed measurements of diffractive vector meson production (HERA, EIC) could significantly enhance our knowledge of fluctuations of the small- x gluon density. They also provide essential input for modeling the dynamics of high-energy pp collisions (RHIC, Tevatron, LHC), where fluctuations play an important role in multijet production and rapidity gap survival. How the concept of parton density fluctuations developed here is affected by the approach to the unitarity limit at high energies remains an interesting problem for further study.

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